

with respect to the last paragraph of Prof. Zeeman's article on Prof. Hale's discovery. The magnetic forces indicated by the splitting up of the lines are not sufficient to produce any direct observable magnetic effect at the distance of the earth.

ARTHUR SCHUSTER.

Simla, October 6.

The Magnetic Disturbances of September 29 and Aurora Borealis.

SOME details of an unusually bright aurora, seen at Omaha, U.S.A., on the night of September 28, local time, may be of interest to the readers of NATURE in connection with the three-hour magnetic disturbance recorded on our magnetograms between 4 a.m. and 7 a.m. of September 29, Greenwich time.

The details come from Father Rigge, S.J., director of the Creighton University Observatory, Omaha. The sky was perfectly clear throughout the night. The aurora "seemed to commence suddenly at 9.50 p.m.," September 28, local time, i.e. at 4.15 a.m., September 29, Greenwich time, when the unifilar magnet at Stonyhurst commenced a rapid westward movement up to 62' of arc at 4.40 a.m., returning more slowly in three sudden steps backward at 5.5 a.m., 5.35 a.m., and 6 a.m., accompanied by minor rapid oscillations.

The aurora was watched for two hours, up to the local midnight, and during this time alternations of the scene were observed between brilliant streamers of various lengths and breadths from a well-defined arch, and a broken-up arch accompanied by drifting luminous patches as of fiery clouds. It would have been interesting to compare the times of these changes with the halting movements of the magnetic needle, but the time was recorded only of the first appearance of the streamers, the smaller lengths of which "seemed to come directly out of the ground," and the noted time agrees closely with that of a single break in the first long and rapid deflection of the needle—a short step-back followed by a rush forward to its greatest elongation. The aurora was again looked for at 5 a.m. of the following morning, when nothing was seen in the still unclouded sky.

It is therefore probable that the auroral display began and ended synchronously with this greater deflection of the needle.

The three-hour wave was, then, followed by the usual rapid oscillations consequent upon a magnetic storm until 2.50 p.m., September 29, G.M.T., when another and a greater storm broke out and lasted until 4.30 of the following morning. At Omaha aurora was again seen at 7.15 p.m., September 29, local time, but in a less favourable sky, which clouded over at 9.15, and showed only by the brightened clouds that the aurora was still active at 10 p.m., when the greater oscillations of the magnets were ending.

WALTER SIDGREAVES, S.J.

Stonyhurst College Observatory, October 21.

A Method of Solving Algebraic Equations.

So far as I can ascertain, the method referred to is not known, at least in its complete form. It is a development of a method described by me in a previous paper ("Verb Functions, with Notes on the Solution of Equations by Operative Division," Proceedings of the Royal Irish Academy, vol. xxv., Sec. A, No. 3, April, 1905), which was reviewed in NATURE of April 25, 1905. I give it here as briefly as possible.

Take, for example, the equation used by Newton to illustrate his method of approximation, namely,

$$x^3 - 2x - 5 = 0,$$

which has one real root, 2.09455. . . . Write the equation in the form $x^3 = 2x + 5$. Select any real number, x_1 ; substitute it for x in the right-hand member of the equation, and then find x_2 from $x_2 = \sqrt[3]{2x_1 + 5}$. Next substitute x_2 for x_1 in the right-hand side of the equation, and find the value of x_3 , and so on. We thus have a series of numbers connected by the equation $x_{n+1}^3 = 2x_n + 5$, and it will be found that whatever number we start with for x_1 , x_n constantly approaches the value of the root. Thus, if we

begin with 11, we have $x_1 = 11$, $x_2 = 3$, $x_3 = 2.2240$, $x_4 = 2.1140$, $x_5 = 2.0975$, $x_6 = 2.0949$, ... Or, commencing with -100, we obtain $x_1 = -100$, $x_2 = -5.7989$, $x_3 = -1.8756$, $x_4 = -1.0768$, $x_5 = -1.0268$, $x_6 = -2.0688$, $x_7 = -2.0907$, $x_8 = -2.0940$, . . .

Again, take the equation $x^3 - 15x - 4 = 0$, which has three real roots, 4 and $-2 \pm \sqrt{3}$, that is, 4, -0.2678, and -3.7321. Write it in the form $x^3 = 15x + 4$, and begin with any number above the limits of the positive roots, say 16. Substitute this for x in the right side of the equation, and proceed as before. Then $x_1 = 16$, $x_2 = 6.2488$, $x_3 = 4.6062$, $x_4 = 4.0124$, $x_5 = 4.0039$, . . . , which is nearly the first root.

In order to obtain the next lower root take for x_1 a number which is a little less than the first root, say 3.9, and substitute it for x , not in the right side of the equation, but in the left side, so that now $\frac{x_1^3 - 4}{15} = x_2$.

Thus we obtain $x_1 = 3.9$, $x_2 = 3.6880$, $x_3 = 3.0558$, $x_4 = 1.6356$, $x_5 = 0.0351$, $x_6 = -0.2666$, $x_7 = -0.2679$, . . . , which is nearly the second root.

For the third root take a number, say -0.3, which is a little less (algebraically) than the second root, and substitute it for x in the right side of the equation, as done for the first root. We thus obtain $x_1 = -0.3$, $x_2 = -0.7937$, $x_3 = -2.0$, $x_4 = -2.9625$, $x_5 = -3.4313$, $x_6 = -3.6203$, $x_7 = -3.6910$, $x_8 = -3.7169$, $x_9 = -3.7267$, . . . , which is nearly the third root.

We can solve the equation in the same manner by beginning with any number, say -5, which is below the limit of the negative roots, and substituting it for x in the right side of the equation; then after finding the lowest root, substitute a greater number for it in the left side of the equation, and so on. We may thus either descend from the highest to the lowest root, or ascend from the lowest to the highest. It is evident that a root is obtained when $x_{n+1} = x_n$, because the equation is then satisfied.

We took the original equations in the forms $x^3 = 2x + 5$ and $x^3 = 15x + 4$, but we may take them also in the forms $x^2 = 2 + 5/x$ and $x^2 = 15 + 4/x$, or in other forms obtained by ordinary algebraic or operative transformations; and the method of solution is the same.

The rule is most easily explained geometrically. Let $f(x) = 0$ be the original equation. Write it in the form $f_2(x) = f_1(x)$, as may usually be done in many ways. Draw the curves $f_2(x) = y$ and $f_1(x) = y$. Then the roots of $f_2(x) = f_1(x)$ are evidently the abscissæ of the points of intersection of the two curves. The procedure adopted above is really as follows. Select any point, x_1 , on the axis of x , and draw a straight line from it parallel to the axis of y , either in the positive or in the negative direction, until it meets the nearer of the two curves—let us say $f_1(x) = y$. From this second point draw a line parallel to y until it meets $f_2(x) = y$. From the third point draw a line parallel to x until it meets $f_1(x) = y$ again, and from the fourth point one parallel to y until it meets $f_2(x) = y$ again, and so on. Then the abscissa of the first and second points is x_1 , of the third and fourth points is x_2 , of the fifth and sixth points is x_3 , and so on, and x_n must generally approach nearer and nearer to the point of intersection of the two curves—that is, to a root of the original equation.

Fig. 1 represents an intersection where the lines drawn according to the rule all lie within the angles formed by the converging curves. In this case, analytically, $x_1, x_2, x_3, \dots$, are all either greater or all less than x , the abscissa of the point of intersection, although they constantly approach it. Fig. 2 illustrates the case where the lines ultimately approach the intersection spirally. Here, analytically, $x_1, x_2, x_3, \dots$ alternately oscillate above and below x , although they constantly approach it. The former, or "staircase" procession, occurs while the differential coefficients of the two curves have the same sign; the latter, or alternating "spiral" procession, while they are of opposite signs.

The staircase procession trends in the same direction as the tangent vectors of the curves if $x_2 - x_1$ is positive, and in the opposite direction if $x_2 - x_1$ is negative. A similar law holds for the direction of rotation of the spiral procession. Thus $x_1, x_2, x_3, \dots$ will increase or decrease, either continuously or alternately, according to whether we have taken x_1 on one or the other of the two curves

$f_2(x)=y$ and $f_1(x)=y$. If we have taken it on the wrong curve they will diverge from the required intersection, as will be apparent from Figs. 1 and 2. The rule to ensure ultimate convergency is that at or near the point of intersection x_1 shall be taken upon the curve which has the numerically lesser differential coefficient. If, at the point of intersection, the differential coefficients are numerically exactly equal, the method fails, as $x_{n+2}=x_n$; but the intersection of the curves will then be at the intersection of the tangents, so that $x=\frac{1}{2}(x_{n+1}+x_n)$ —Fig. 3. It often happens, if we have taken x_1 at random, that the succeeding terms of the series are at first irregular, but afterwards converge.

If at any stage in the analytical process a term becomes unreal, this means that the corresponding line drawn from one of the curves cannot intersect any branch of the other curve. We must then start again with another value of x_1 .

The successive terms may appear to converge for a time and may then diverge. This indicates the position of a pair of imaginary roots (Fig. 4). Compare, for instance, $x^2=13x-42$, of which the roots are 6 and 7, with $x^2=13x-43$, of which the roots are imaginary.

Convergency is slowest when the differential coefficients of the curves at the required intersection are nearly equal, numerically, to each other, especially if both are also nearly equal to ± 1 . It is quickest when their numerical difference is greatest. When we have arrived near enough

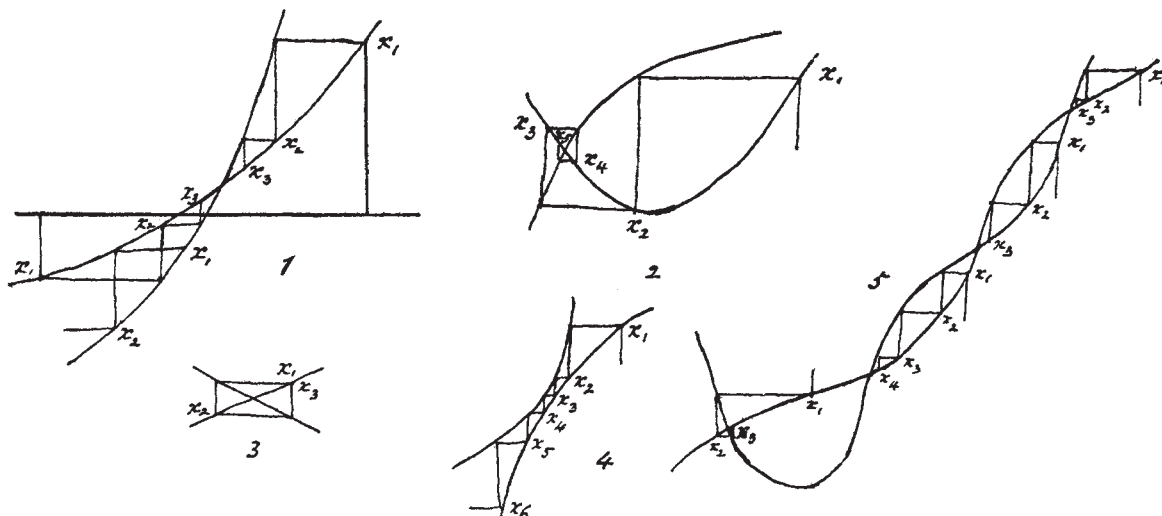
ever, this can be avoided by finding special roots from other forms of the equation. For example,

$$x^3-3x^2-2x+5=0$$

has three roots, 3.128, 1.202, and -1.330, but they can be calculated more easily from the form $(x-1)^3=5x-6$. One or two of the roots of a complete rational integral equation may frequently be obtained almost at once by dividing the equation by x^{-1} , and putting it in the form $x=a+b/x+c/x^2$ Generally, the first terms of the series x_1, x_2, x_3 . . . may be estimated mentally, exactness being unnecessary until we approach near to a root.

The rule, therefore, has the advantage of being very easily remembered, of giving, theoretically, all the roots in succession, and of leading, almost automatically, to at least one or two solutions. Hitherto it has been considered from the geometric and arithmetic side; I will now try to indicate briefly its operative and algebraic forms.

We have evidently to do with repeated operation, which is best expressed by the algorithm of "verb functions" as described in my paper referred to. This algorithm is based on the fact that ϕ^0 , where the index refers to operative and not algebraic involution, cannot possibly be the equivalent of numerical unity (as generally held), but is equal to *operative unity*. I denote this by the symbol B (for base). When substituted for the argument in any expression, B converts that expression into one which



to the destination, further work may often be abbreviated by assuming that the intersection of the curves nearly coincides with that of the tangents. Thus if x_1 and x_2 are successive convergents on the curves $f_1(x)=y$ and $f_2(x)=y$ respectively, then, approximately,

$$x = \frac{x_1 f_1'(x_1) - x_2 f_2'(x_2)}{f_1'(x_1) - f_2'(x_2)}.$$

In the case of the equation $x^3=15x+4$ the roots were obtained successively by taking x_1 alternately, first on one curve, $15x+4=y$, and then on the other, $x^3=y$. This can be done very frequently, but sometimes one of the curves makes such a bend between two intersections that, by the rule already given to ensure convergency, x_1 may have to be taken on the same curve for two roots in succession. By plotting the curves roughly on paper it is generally easy to see at a glance how best to commence and conduct the process (Fig. 5).

As is evident from the geometrical interpretation, the method is by no means restricted to rational integral equations. The two curves $f_2(x)=y$ and $f_1(x)=y$ may be any we please, provided only that we can obtain $x=f_2^{-1}(y)$, or at least can evaluate it for different values of y , and it is generally easy to put the original equation $f(x)=0$ in such a form that this can be done; but in order to find some of the roots it may be necessary to take x_1 on the other side of the equation, which requires us also to obtain x from $x=f_1^{-1}(y)$, which may be difficult. Generally, how-

denotes an action, not a substantive. Thus $a+b\beta+c\beta^2$ is the action performed on x in order to convert it into $a+bx+cx^2$.

Now let $x^n+ax^{n-1}+bx^{n-2} \dots =k$, the number of terms being unlimited. Then

$$x = [\sqrt[n]{k - a\beta^{n-1} - b\beta^{n-2} \dots}]x,$$

where the square brackets denote that the expression contained within them operates on the following matter, and is not multiplied into it. Thus x on the left side of the equation is the result of an operation performed on itself. Similarly, x on the right side of the equation is the result of the same operation performed on itself as many times as we please. Hence we obtain the identity

$$x = [\sqrt[n]{k - a\beta^{n-1} - b\beta^{n-2} \dots}]^q x,$$

where q denotes operative involution and may be any integer, positive or negative, to infinity. Now the expression on the right can be developed with the aid of the multinomial theorem by successive substitution, according to the common algebra of verb functions, and we obtain

$$x = k^{\frac{1}{n}} - \frac{1}{n} a - \frac{1}{1} \left\{ \frac{1}{n} b + \left(\frac{1}{n} \right)^{(2)} \frac{a^2}{2!} \right\} k^{-\frac{1}{n}} - \frac{1}{2} \left\{ \frac{2}{n} c + \left(\frac{2}{n} \right)^{(2)} ab + \left(\frac{2}{n} \right)^{(3)} \frac{a^3}{3!} \right\} k^{-\frac{2}{n}} \dots$$

+ terms containing x .

When this series proceeds to infinity the terms containing x , under certain conditions, vanish. So also they vanish if any proximate number is substituted for x in them. We are thus left with k , on the left side, equated to a series containing only k and the coefficients $a, b, c, \dots$. This explains why we may start the process described at the beginning of this paper with *any* number (under certain limitations) for x , because, whatever that number may be, it is gradually rendered negligible by the successive operations.

The series has been already studied to some extent in the paper referred to, and has been used for solving equations. Its coefficients are simply those of the multinomial theorem with some modifications. As it has n values depending on the n values of $\sqrt[n]{k}$, we may suppose that these values are the n roots of the original equation, though we may not be able yet to evaluate all of them. This has been proved in the previous paper to be the case, because the sums of the products of the values taken one, two, three . . . times together are equal to the successive coefficients of the original equation with the proper signs. Hence there are some reasons for thinking that the series theoretically constitutes the general *transcendental* solution of the equation of the n th degree. How far this is really the case must be discussed more fully on another occasion, together with details and developments of the method outlined above.

The method is not the same as the methods of approximation of Newton, Lagrange, and Horner. The well-known ascending power series for the reversion of a function, and cases in which certain repeated operations (such as continued fractions) converge to a root of an equation, thus solving certain functional and difference equations, are only particular instances of the above theorem.

RONALD ROSS.

The Nature of X-Rays.

In a letter to NATURE of July 30 Prof. Bragg tries to show that his neutral-pair theory of X-rays may form the basis of an explanation of the secondary X-ray phenomena which I briefly summarised in an earlier letter (May 7). He, however, neglects the consideration of so much important evidence that I cannot attempt to reply in detail. In reply to his discussion of statements (3), (6), and (5), I need only state that he has confused two distinct types of secondary X-radiation, and that his statement of Mr. Crowther's results is inaccurate when applied, as he applies it, to the scattered radiation alone. (May I also be permitted, in passing, to point out that both the general results attributed to Mr. Crowther had been published by the writer previous to the publication of Mr. Crowther's paper?)

Again, Prof. Bragg has evidently overlooked the work to which I referred in statements (7), (8), and (9). The evidence which I put forward for consideration was not the older work of M. Sagnac, Dr. Walter, and Mr. Adams which Prof. Bragg discusses, but the results of experiments by Mr. Sadler and myself on homogeneous beams of X-rays, which have not yet been published in full, though preliminary notices had appeared in NATURE. The paper giving an account of this work was read before the London Physical Society on June 12. Prof. Bragg, as a consequence, does not discuss the points with full knowledge of experimental facts.

Of the three remaining points, one—the polarisation of a primary beam (1)—is not discussed, because Prof. Haga has been unable to verify it by a much cruder method than that originally employed. It is nevertheless a physical fact.

Finally, two results—the polarisation in scattered radiation (4) and the equality in the penetrating powers of primary and secondary (scattered) rays (2)—which appear possible to Prof. Bragg on the neutral-pair theory, require assumptions which, to my mind, are extremely doubtful. On the other hand, many of these results were foretold on the ether pulse theory, and, indeed, they all find an easy explanation on this theory, as I believe Prof. Bragg will readily admit when he has become fully acquainted with

the experiments. For a fuller discussion I can, unfortunately, only refer to two unpublished papers, both of which, however, are in the press. These are the one already referred to and one which will appear in the forthcoming number of the "Jahrbuch der Radioaktivität und Elektronik."

In reply to Prof. Bragg's contention, may I add that the phenomena involving radiation of only one kind—X-radiation—to me appeared simpler than those involving two—X and β radiations?

Liverpool, August 8.

CHARLES G. BARKLA.

It is, of course, true that my letter (dated June 5) to which Dr. Barkla refers was written before I had had the opportunity of studying Dr. Barkla's latest results. A portion of my argument was based on his earlier work, and may need a little alteration in consequence. I have myself found by recent experiment that his older statements needed amendment. For example, the emergence and incidence secondary Röntgen radiations differ both in quality and quantity; the former is sometimes far greater than the latter.

May I take this opportunity of correcting a statement in a letter of mine which appeared in NATURE of July 23? As pointed out in an addendum to a recent paper contributed by Dr. Laub to the *Annalen der Physik*, I have been wrong in supposing that Dr. Wien still maintains that the energy of the secondary kathode ray is drawn from the energy of the atom. Had I understood Dr. Wien correctly, I should certainly not have taken so much pains to disprove a theory which he had already abandoned.

W. H. BRAGG.

The University of Adelaide, September 17.

The Supposed Inheritance of Acquired Characters.

DR. FRANCIS DARWIN, in his presidential address before the British Association, writes as follows:—

"Fischer showed that when chrysalids of *Arctia caja* are subjected to a low temperature a certain number of them produce dark-coloured insects; and further that these moths mated together yield dark-coloured offspring. This has been held to prove somatic inheritance, but Weismann points out that it is explicable by the low temperature having an identical effect on the colour-determinants existing in the wing-rudiments of the pupa, and on the same determinants occurring in the germ-cells."

It occurs to me that still another explanation is possible to cover at least some such cases. In discussing various types of latency, Dr. Shull (*American Naturalist*, July) has recently defined as "latency due to fluctuation" those cases (of which many are known) in which the special characters of a race do not appear except under suitable conditions. Following this idea, it is possible to think of the dark *Arctia caja* appearing after exposure to cold as representing a variation which possessed an inherent tendency to darkness not exhibited under more ordinary conditions. Indeed, this must have been the case, since only "a certain number" were affected. Given such a variation, it is not unreasonable to suppose that when examples were mated together the tendency would be so emphasised as to appear under normal temperatures, thus producing an apparent case of the inheritance of acquired characters.

T. D. A. COCKERELL.

University of Colorado, October 7.

Determination of Sex: a Correction.

MAY I correct a slip in your report of "Zoology at the British Association" (NATURE, October 22, p. 647)? The cinnamon canaries resulting from the mating green hen x cinnamon cock are all *females*, not males, as there accidentally stated. The point is critical in the interpretation of that curious case.

W. BATESON.

October 26.